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THE ALLOCATION OF
AVAILABILITY PARAMETERS--
REPAIR TIMES AND FAILURE RATES

RESEARCH REPORT

Presented in Partial Fulfillment of the Requirements
For the Degree Master of Engineering, Industrial
Engineering Department of Texas A&M University

By

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1970

ABSTRACT

This paper develops a cost-based procedure for allocating the availability parameters, repair times and failure rates, to the various components that make up a system. The allocation problem is handled as a cost minimization problem, subject to the constraint of meeting the system availability requirement. A Lagrange multiplier technique is employed to obtain the solution. An example problem is stated and solved in the context of a computer program developed to perform the allocation procedure.

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CHAPTER I

INTRODUCTION TO THE PROBLEM

The requirements for new system procurements are more and more being directed toward the attainment of adequate overall system performance. Today, more is required of a new system than the ability to pass a single performance test; system requirements are more detailed, more complex, and more encompassing than in the past. As system requirements become more involved and more complex, it becomes increasingly necessary to speak of the parameters of total system performance in definite, quantitative terms.

In discussing the requirements for total system performance, the Martin-Marietta Maintainability text (12)* states "...the overall objective is availability, to which reliability and maintainability contribute in varying degrees, depending on the system or equipment being developed and on the conditions under which it will be used." Although it is an over-simplification to attempt to characterize a system's performance by a single number, it is still necessary to begin to think in quantitative terms about total system performance. Historically, the most common term used to describe total system performance has been availability.

If system availability can be thought of as a function of reliability and maintainability, then early in the system design, the definite quantitative relationships between these parameters should be analyzed carefully.

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*Numbers in parentheses refer to list of references at the end of the paper.

Furthermore, the costs associated with achieving a particular availability requirement should be detailed for each system component. In order to coordinate the efforts of different groups concerned with different system characteristics, and to eliminate the hazards of guesswork in achieving the overall system requirement, it is necessary to establish a procedure for determining the detailed specifications for the various components that make up a system. The process of assigning availability parameters to individual components to insure the attainment of the system availability goal is termed availability allocation.

Balaban and Jeffers (3) have listed some of the values to be derived from an effective detailed allocation program. Some of these are summarized below:

a) Definite quantitative availability requirements force contractors to consider availability parameters along with system characteristics such as performance, weight, and cost.
b) Allocation focuses attention on the relationships between component, equipment, subsystem, and system availability, leading to more complete understanding of the basic reliability-maintainability problems inherent in the design.
c) Requirements determined through an allocation procedure are more realistic, consistent, and economical than those obtained through subjective or haphazard methods, or those resulting from crash programs initiated after bitter field experiences.

Generally, reliability-maintainability requirements are imposed, based on field need, past experience, combinations of these and other criteria. The problem that is solved in this paper assumes that an optimal availability requirement has been determined, and the paper is concerned with the allocation of parameters to achieve that requirement.

The allocation of system availability involves solving the basic inequality

$$f\,(A_1, A_2, A_3, \ldots A_n) \geq A^*,$$

where A* is the system availability requirement, A_i is the allocated availability for i^{th} unit, and f is the functional relationship between unit and system availability. The above inequality has an infinite number of solutions if no restrictions are placed on the allocation. The problem is to establish a solution procedure which yields a unique or limited number of solutions, by which reasonable and consistent availability parameters may be determined. Generally, the only parameters that are considered significant contributors to the availability are the component failure rates and repair times. Cost functions associated with these factors are discussed in Chapter IV. The approach used in this paper is to minimize the cost of availability improvement subject to the constraint of meeting the system availability goal. Mean repair times and failure rates are allocated to the system components based on the relative costs of improving the repair times and the failure rates.

The solution of this problem for the availability parameters, repair times and failure rates, where availability is defined by the expression

$$A = \frac{MTBF}{MTBF + MTTR}$$

where MTBF is the mean time between failures and MTTR is the mean time to repair, is accomplished under the assumptions of constant failure rates (exponential time-to-failure densities) and independently failing components. Also, a series configuration is assumed in which all components are necessary for proper system functioning. The limitations and implications of these assumptions are discussed in Chapter III.

In order that this paper be as widely applicable as possible, a general solution procedure was followed to the extent feasible. Therefore, a solution set of differential equations is expressed under only the assumptions listed above. This solution set may be used with any applicable cost functions. One specific cost function is considered for

detailed analysis and solution in this paper. However, by simply substituting any differentiable cost functions into the solution equations, a set of $2n + 1$ equations in $2n + 1$ unknowns would be obtained, where n is the number of system components being considered. Of course, one still is faced with the problem of solving the simultaneous set of equations for the allocated failure rates and repair times. This presents no serious drawback, however, since numerical computerized techniques are readily available for the solution of such systems of equations. The methods used in this paper should prove helpful in providing solution techniques for similar cases.

The cost function used takes the form of a reciprocal relation (a decreasing hyperbolic form) dependent on the amount of change of failure rate and/or repair time necessary to improve the availability to the required level. The cost function is assumed to take the same form for each component with only cost constants differing. This allows the model to be applied in cases where only rough or relative factors can be established for the cost of improving failure rates or repair times. The development of this model is given in Chapter IV. The allocation model developed with the specific cost function appears to be applicable to any system where the constant failure rate and series assumptions can be applied with reasonable accuracy.

The allocation problem is approached in this paper from two basic points of view. First, it is assumed that a particular design has been proposed or developed that falls short of the system availability requirement. The allocation procedure is developed to determine the optimal method (least cost method) for improving system availability to the required level. In the second case, it is assumed that during the conceptual stage of system development when no design has been formalized, an

allocation based on estimated costs of reliability and maintainability development can give the optimal set of component parameters that should be established as system goals.

It should be noted that the allocations are based only on mean-time-between-failures (the inverse of the failure rate, λ, for the exponential) and mean repair times. No assumptions are made concerning the distributions of the times to repair. In this respect, one additional factor that has been considered is the imposition of an additional constraint on the allocation. This constraint is a maximum allowable repair time that cannot be exceeded under any circumstances. The point to be made is that, although mean system repair time might appear to be acceptable, it is possible that a low failure rate component with an enormous repair time could produce disasterous results when a failure did occur. This point is discussed in Chapter V.

In Chapter III the general allocation technique is described, and limitations and restrictions are discussed. The specific cost function is developed in Chapter IV; the solution to this case is obtained and additional forms of cost functions are discussed. In Chapter V an example problem is stated and solved, and computer application of the allocation technique is discussed. The summary of this work, conclusions, and recommendations for further study are given in Chapter VI.

A brief literature survey is discussed in Chapter II, introducing pertinent references to reliability-maintainability areas, specifically to allocation-type problems.

CHAPTER II

LITERATURE REVIEW

General

This chapter reviews some of the available literature dealing with allocation and allocation techniques, reliability-maintainability costs and cost relationships, maintainability prediction, resource allocation, and availability assessment.

It was found that large amounts of work have been accomplished in reliability allocation; some of the more important and interesting approaches are detailed below. However, very limited work has been carried out in the field of maintainability allocation. There appear to be two obvious reasons for this discrepancy: first, there simply has not been sufficient time in the brief history of maintainability for such development to occur, and secondly, there is an added complication of being concerned with two parameters, repair time (or repair rate) in addition to the failure rate which is considered by itself in reliability allocation. Since these two parameters are the basic components of availability, this paper considers the allocation of availability parameters, failure rates and repair times.

Reliability Allocation

At this point a brief description of some of the current reliability allocation techniques will help to establish a background for the development of this paper. Although some of these techniques appear to be well

developed and complete, they are concerned only with the allocation of reliability. The extensions to the availability area, which is concerned with the parameters of reliability and maintainability, have not been carried out. The ARINC Reliability text (2) proposes an allocation method based on predicted failure rates assuming exponential density and series configuration (all components necessary for operation). A weighting factor,

$$w_j = \frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j}$$

where λ_j is the failure rate of j^{th} component in system of n components, is calculated for each component. The "portion" of the system failure rate allocated to each component is then determined as $\lambda_j = w_j\lambda^*$ where λ^* is the desired system failure rate. Obviously, this method considers only the predicted component failure rates and allocates to each component an improved, weighted failure rate which will allow the reliability requirement to be met.

A technique developed by AGREE (18) follows the same assumptions as the ARINC method; however, this report considers an importance factor for each component such that the necessity of a particular component for overall system operation is considered. The importance factor is a conditional probability of system failure given a specific component failure. Although additional refinements have been applied to the above techniques (2, 3), these methods do not consider the cost or complexity of improving reliability to meet a system requirement. These methods are often termed basic allocation techniques--that is, the computations depend only on the parameter values themselves, and cost, weight, complexity, or time considerations in terms of improving a piece of equipment are not considered.

A technique developed by Neuner and Miller (14) involves a trade-off between a variable such as cost or weight and component reliability. System reliability is defined by $R_s = f(R_1, R_2, \ldots, R_n)$ where each R_i is a function of the trade-off variable (V_i) which is subject to physical or other constraints. The problem becomes:

$$\text{maximize} \quad R_s = f\left[R_1(V_1), R_2(V_2), \ldots, R_n(V_n)\right]$$

$$\text{subject to:} \quad V_s = g(V_1, V_2, \ldots, V_n) \quad ,$$

where V_i is the amount of resource allocated to the i^{th} component and V_s is the total amount of resource available. Neuner and Miller solve the problem by the method of Lagrange multipliers for the case where the series assumption and additive resources (no duplication) apply. Fyffe, Hinds, and Lee (10) have used a Lagrange multiplier solution and an iterative computational algorithm to perform reliability maximization according to two constraints—cost and weight. Their solution was based on selecting design alternatives to meet the allocated reliability requirement.

Albert (1) approached the reliability allocation problem by defining a function which was a measure of the effort required to increase subsystem reliability to an improved level. He minimized this "effort function" subject to a system reliability requirement for the case in which each component had the same effort function and the reliability of the system was the product of the component reliabilities (series assumption). Further assumptions for this solution restricted the form the effort function could assume.

A procedure developed by Weir (21) provides a relatively straight-forward method for optimizing reliability with respect to cost for specific system reliability models. He simply considers the cost of buying an

increment of improvement of reliability as a constant and develops the procedure for a model which is more complicated than the simple series model. This report also discusses in some detail computer application to specific models.

Reliability-Maintainability Relationships and Allocations

A basic maintainability allocation technique is presented in the recent text by Blanchard and Lowry (6). This technique is also presented in the Martin-Marietta prepared AMCP-705-1 (12). The technique calculates system availability according to the definition,

$$A = \frac{MTBF}{MTBF + MTTR} ,$$

which is the mean time between failures divided by total time. This is the usual definition for inherent availability (2); that is, only active repair time and mean time between failures are considered. The Martin-Marietta technique calculates system repair time by summing component repair times, each multiplied by its expected fraction of occurrence,

$$f_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} .$$

Thus, given a system availability requirement and component failure rates, the system repair time goal can be determined. If estimated or predicted component repair times are inadequate to meet the availability requirement, then general recommendations are given as to which components to improve. Ideally, components contributing the highest percentage to the total failures require a low repair time, and those with low contributions can have higher repair times. However, quantitative relationships for these trade-offs have not been developed. In addition, this maintainability allocation technique considers only the predicted repair times and failure

rates; no attempt is made to define the effect of the changes on cost of availability or availability improvement.

Eimstad (7) presents an availability (effectiveness) allocation method which depends on complexity factors with checks to see that repair rate and failure rate extremals are not exceeded. He also discusses a cost-effectiveness or design trade-off analysis which is graphically solved for certain specific effectiveness models.

Several additional papers have been found which deal with aspects of reliability-maintainability problems that were considered pertinent to the work carried out in this paper. Although these reports are related to the field of interest of this paper, they do not attempt to solve specific availability allocation problems. Muth (13) considers a system defined by a reliability network and the failure rate and repair rate of each component. He develops a "downtime" distribution for the case in which both repair time and time to failure are exponentially distributed. This work is not extended to consider allocation. Bazovsky (4) has written several articles in this general area--especially in consideration of applications to mechanical systems.

Reich and Miller (16) have used linear and dynamic programming approaches to perform reliability and maintainability cost trade-offs. In this regard, many of the well-established linear and dynamic programming (5) and resource allotment or allocation techniques could be applied to reliability and maintainability design problem areas. Examples of this type of application are given by Todaro (19). Todaro considers a dynamic programming approach for determining repair or replace maintenance policies and optimal module sizing.

In summarizing this chapter, it is obvious that considerable work has been carried out in the field of reliability allocation in addition to basic

work in maintainability areas related to the allocation problem. However, the need still exists to establish techniques for the simultaneous, cost-based allocation of the parameters of total system performance. Therefore, this paper attacks the problem of allocating the availability parameters, repair times and failure rates, to the components of a system, based on cost considerations associated with availability improvement.

In Chapter III the development of the allocation problem introduced in the first chapters of this paper is begun. The general allocation technique is described and developed, and limitations and restrictions are discussed.

CHAPTER III

BASIC DEVELOPMENT OF THE ALLOCATION PROBLEM

General

With the conceptual development of a new or improved operating system, the planning and organizing groups usually establish basic guidelines as to the system's characteristics of operation. The mission or job that is to be performed is described in as much detail as possible, and acceptable methods for attacking the job are discussed. Primary among the concerns of these groups are the consequences or price of failure to adequately perform the mission. Thus, it becomes necessary to ensure that a system is capable of performing properly, and furthermore, that it is capable of performing properly a large percentage of the times it is called on to operate. This last aspect of system operating characteristics, the percentage or fraction of total time that a system is in proper operating condition, is termed availability.

The availability of any system can be thought of as a function of the reliability and maintainability of that system. This can be represented by the following equation.

$$A = f(R, M) \tag{1}$$

where A is the system availability, R is the system reliability and M is the system maintainability. The prime elements of reliability and maintainability are the system mean-time-between-failures and the mean-time-to-repair respectively. Thus the availability can be concisely defined as a

function of these two parameters. Furthermore, the availability is mathematically defined as the fraction of total time that a system is in operating condition. That is,*

$$A = \frac{MTBF}{MTBF + MTTR} \tag{2}$$

where MTBF is the mean-time-between-failures in hours and MTTR is the mean-time-to-repair in hours.

If one considers a series system made up of components having constant failure rates, then the system failure rate can be written as the sum of the individual failure rates if all components fail independently. This is true under these conditions, since the presence of other components does not affect the characteristics of any particular component, and the failure of one component causes system failure. Thus,

$$\lambda_s = \lambda_1 + \lambda_2 + \ldots \lambda_n$$

$$\text{and } MTBF = \frac{1}{\sum_{i=1}^{n} \lambda_i} \quad . \tag{3}$$

The system repair time can be thought of as the weighted mean of the component repair times (M_i) where the weighting factors are the ratios of the component failure rates to the total failure rate. Thus, if each component's repair time is multiplied by its expected fraction of occurrence and the resulting values summed, then the expected system repair time is obtained as

*It should be noted that this definition is the definition that most authors call inherent availability; the mean-time-to-repair would not be taken to include logistics or administrative downtime, but simply the actual repair time of the system given adequate repair facilities and spare parts. Thus, this definition represents the "best case" or what is often called the designed-in availability.

$$\text{MTTR} = \frac{\lambda_1}{\lambda_s} M_1 + \frac{\lambda_2}{\lambda_s} M_2 + \ldots + \frac{\lambda_n}{\lambda_s} M_n$$

$$= \frac{1}{\lambda_s} \sum_{i=1}^{n} \lambda_i M_i = \frac{1}{\sum_{i=1}^{n} \lambda_i} \sum_{i=1}^{n} \lambda_i M_i \quad . \tag{4}$$

Substituting the expressions for MTBF and MTTR into the original availability definition (Equation 2) yields

$$A = \frac{1}{1 + \sum_{i=1}^{n} \lambda_i M_i} \tag{5}$$

for the availability of a system in which the components exhibit constant failure rate, the components fail independently, and all components are necessary for system operation. The limitations and implications of these assumptions are discussed in the following section.

Limitations Due to Assumptions of Constant Failure Rate and Independent Components All Necessary For System Operation

The assumption of constant failure rates, or stated otherwise, the assumption of an exponential time to failure density, dictates that the failure rate be constant throughout the life of the equipment. In addition to being necessary to analytically solve most of the system relationships, this assumption appears quite reasonable when one considers the allocations of 'designed-in' characteristics. The point to be made is that a design is a definite, concrete plan for a system to perform a certain function; therefore, definite specifications must be established for the system to ensure that it can perform the expected operational functions. These specifications must be established for some period in the equipment life. In other words, the design could not be based on the failure rate during

the first year since gross underdesigning would probably occur due to the low failure rate. On the other hand, if the failure rate at the twentieth year were used, then overdesigning, overstockage of spares in the early years, and other related costs would be overwhelming.

The alternative would be to consider a changing failure rate and keep spares and repair facilities current with the failure rate of the system. The ramifications of this approach are quite significant. For instance, if one system failed, then the replacement would exhibit a completely different failure pattern than the other like items in the operational system. As additional items failed, keeping track of combinations of the older failure rates and the newer failure rates would be extremely difficult if not impossible. Since most well-cared for items exhibit a fairly long constant failure period (2), it seems reasonable that the exponential would not only be a good, relatively straightforward approximation for a system, but would probably be quite realistic when considering the total life of a large group of operating systems.

The series assumption is somewhat restrictive in that no provision can be made for operation in a degraded mode. However, by careful definition of the 'components' that make up a system, it appears that reasonable approximations to a series system could be obtained. For instance, an item with three independent, necessary components and one redundant component for the first item, could be defined by calling it a three component system, where the first component's reliability was the reliability obtained by combining the primary and redundant element into one component. Similarly, higher order components can be rearranged and combined until a series configuration could be approximated. It should be pointed out that the combination of elements with constant failure rates does not result in new components with constant failure rates, and therefore, the constant failure

rate assumption must be reanalyzed. Care should be taken to ensure that proper allowances are made for cases of this nature and that the best possible estimates are made for failure rates and repair times.

The assumption of independently failing components is usually made in reliability discussions. This assumption is subject to some question since, in most any mechanical or electrical equipment, the failure of one component could damage or degrade other components. However, by the judicious combination of components which appear to fail dependently, approximations of the overall failure rate can be made. It should be noted, however, that this assumption, like the others, does somewhat restrict application of the models developed in the paper, and full consideration should be given to the assumptions whenever the model is applied to an allocation problem.

Basic Development of Cost Relationships

The allocation of availability parameters to the individual components of a multi-component system involves making the decisions as to which components to consider for availability improvement and to which characteristics, repair time and/or failure rates, improvement effort should be applied. This section discusses the basic development of these decision criteria.

It is assumed for this development that a system configuration has been proposed which falls short of the availability goal established as the system requirement. That is, $A < A^*$, where A^* is the system availability requirement. Then, from Equation 5*

$$\frac{1}{1 + \Sigma\lambda_i M_i} < A^* \quad . \tag{6}$$

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*All summations are from 1 to n unless otherwise specified.

In order to simplify the development that follows, it is first assumed that the reliability of the system is at its highest achievable level. Thus, only the repair times (maintainability) can be improved. Then, the opposite case is considered in which only the failure rates can be improved. Finally, the two cases are combined to give the overall optimal set of improvements

Case 1: Reliability Fixed at the Highest Achievable Level

Let $\hat{M}_i$ be the allocated repair time of the i^{th} component such that Inequality 6 becomes

$$\frac{1}{1 + \Sigma\lambda_i \hat{M}_i} = A^* \quad . \tag{7}$$

Rewriting, Equation 7 yields

$$\Sigma\lambda_i \hat{M}_i = \frac{1}{A^*} - 1 \quad . \tag{8}$$

Since, M_i is the original component repair time, then

$$\hat{M}_i = M_i + \Delta M_i \tag{9}$$

where ΔM_i is the improvement in repair time for the i^{th} component. For any component, ΔM_i can be zero, negative, or positive. The case in which a ΔM_i becomes positive indicates that the optimal solution is obtained by decreasing the repair time of some components while increasing the repair time of the component with positive ΔM_i. In the development of Chapter V, two alternatives are considered in this case. First, the positive change is set equal to zero and a new optimum is determined since, normally, one would not purposely increase the repair time of a component. Secondly, the change is considered to have occurred when one component is improved at the

expense of another. For instance, if the packaging arrangement called for one component to be closer to the surface for easier access while another component was farther inside the package, then it might be possible to simply exchange the two components, thereby increasing the repair time of the component with positive ΔM_i and decreasing the repair time of the component with the negative ΔM_i. In any event, the consideration of both cases would serve to increase one's understanding of the design under consideration. In all cases the results of the allocation procedure should be interpreted as guidelines, rather than as absolute rules to follow, in achieving the availability requirement.

Substituting the expression for $\hat{M}_i$, Equation 9, into Equation 8, separating the summations and simplifying, Equation 10 is obtained as

$$\Sigma\lambda_i \, \Delta M_i = \frac{A - A^*}{A^* A} \quad . \tag{10}$$

In order to express the cost of improving the repair time, the cost equation

$$C_i = g_i \, (M_i, \Delta M_i)$$

is defined where g_i is the functional relationship for the cost of decreasing the repair time M_i by ΔM_i. For the entire system, the total cost (TC) is given by

$$TC = \Sigma g_i \, (M_i, \Delta M_i) \quad . \tag{11}$$

No definite forms of g_i are considered in this chapter.

The allocation problem is thus expressed as the problem of minimizing the total cost of availability improvement (in this case, only the repair time improvement) subject to the constraint of meeting the system

availability requirement. Mathematically, this means:

$$\text{minimize} \quad TC = \Sigma g_i \, (M_i, \Delta M_i)$$

$$\text{subject to:} \quad \Sigma\lambda_i \, \Delta M_i = \frac{A - A^*}{A^*A} \quad .$$

Case 2: Maintainability Fixed at the Highest Achievable Level

In this case the availability requirement is expressed as a function of the achieved repair times and the allocated failure rates,

$$A^* = \frac{1}{1 + \Sigma\hat{\lambda}_i \, M_i} \tag{12}$$

where

$$\hat{\lambda}_i = \lambda_i + \Delta\lambda_i \tag{13}$$

and $\Delta\lambda_i$ is the failure rate improvement for the i^{th} component. As before, it is possible that the optimal solution would show a positive value for $\Delta\lambda_i$, such that an increase in λ_i is indicated. This situation is handled in the same manner as that described for the repair time case.

Substituting for $\hat{\lambda}_i$ in Equation 12 and simplifying yields

$$\Sigma\Delta\lambda_i \, M_i = \frac{A - A^*}{A^*A} \quad . \tag{14}$$

Defining a cost function, $h_i \, (\lambda_i, \Delta\lambda_i)$, for the cost of changing the failure rate, λ_i, by $\Delta\lambda_i$, the total cost of availability improvement for Case 2 becomes

$$TC = \Sigma h_i \, (\lambda_i, \Delta\lambda_i) \tag{15}$$

and the allocation problem is the minimization of Equation 15, the total cost, subject to meeting the constraint of the availability requirement, Equation 14.

Case 3: Repair Times and Failure Rates Can Be Improved

If the system availability requirement is expressed as a function of both the allocated repair times and allocated failure rates, the equation for the availability requirement becomes

$$A^* = \frac{1}{1 + \Sigma \hat{\lambda}_i \hat{M}_i} \quad . \tag{16}$$

Substituting Equations 9 and 13 into Equation 16, gives

$$A^* = \frac{1}{1 + \Sigma (\lambda_i + \Delta\lambda_i) (M_i + \Delta M_i)} \quad .$$

Simplification yields the equation

$$\Sigma M_i \, \Delta\lambda_i + \Sigma \Delta M_i \, \lambda_i + \Sigma \Delta\lambda_i \, \Delta M_i = \frac{A - A^*}{A^* A} \quad . \tag{17}$$

The total cost is expressed as the sum of the costs for improving the failure rates and the repair times, such that the allocation problem becomes:

$$\text{minimize} \qquad TC = \Sigma h_i (\lambda_i, \Delta\lambda_i) + \Sigma g_i (M_i, \Delta M_i) \tag{18}$$

$$\text{subject to:} \quad \Sigma M_i \, \Delta\lambda_i + \Sigma \Delta M_i \, \lambda_i + \Sigma \Delta\lambda_i \, \Delta M_i = \frac{A - A^*}{A^* A} \quad . \tag{17}$$

General Solution of the Allocation Problem

The general solution to the allocation problem expressed in Case 3 is obtained by a Lagrange multiplier technique. Solutions to Cases 1 and 2 are simply special cases of 3, and although the solutions are not stated explicitly, either solution could be obtained by considering the set of parameters of interest, failure rates or repair times, to be constants, such that all the $\Delta\lambda_i$ or ΔM_i would be zero.

If the constraint equation is written to be equal to zero, that is

$$\Sigma M_i \, \Delta\lambda_i + \Sigma \Delta M_i \, \lambda_i + \Sigma \Delta\lambda_i \, \Delta M_i - \frac{A - A^*}{A^* A} = 0 \quad , \tag{19}$$

then the Lagrangian can be written as

$$\Lambda = \Sigma h_i \, (\lambda_i, \Delta\lambda_i) + \Sigma g_i \, (M_i, \Delta M_i)$$

$$+ K \left(\Sigma M_i \, \Delta\lambda_i + \Sigma \Delta M_i \, \lambda_i + \Sigma \Delta\lambda_i \, \Delta M_i - \frac{A - A^*}{A^* A}\right)$$

where K is the Lagrange multiplier.

Taking the derivatives and setting them equal to zero yields the following set of equations:

$$\frac{\partial \Lambda}{\partial \Delta\lambda_i} = \frac{\partial h_i \, (\lambda_i, \Delta\lambda_i)}{\partial \Delta\lambda_i} + K M_i + K \, (\Delta M_i) = 0 \quad ,$$

for $i = 1, 2, \ldots, n$

$$\frac{\partial \Lambda}{\partial \Delta M_i} = \frac{\partial g_i \, (M_i, \Delta M_i)}{\partial \Delta M_i} + K\lambda_i + K \, (\Delta\lambda_i) = 0$$

for $i = 1, 2, \ldots, n$

and

$$\frac{\partial \Lambda}{\partial K} = \Sigma M_i \, \Delta\lambda_i + \Sigma \Delta M_i \, \lambda_i + \Sigma \Delta\lambda_i \, \Delta M_i - \frac{A - A^*}{A^* A} = 0 \quad .$$

The above set of equations contains 2n + 1 unknowns, $\Delta\lambda_1$, $\Delta\lambda_2$, ..., $\Delta\lambda_n$, ΔM_1, ΔM_2, ..., ΔM_n, K, and since there are 2n + 1 equations, a unique solution should be obtainable if definite forms of the cost functions are inserted. It should be pointed out that the solution set of equations is general in that any differentiable cost functions could be inserted and

solutions obtained. However, the assumptions involved in the development (constant failure rates, independently, failing components, and series configuration) should be analyzed in the particular application under consideration.

In Chapter IV particular cost relations are discussed and one form of cost equation is considered for detailed analysis and solution. The methods of solution used in Chapter IV will offer guidelines for solution techniques that could be applied to other cost functions.

CHAPTER IV

COST-BASED SOLUTION OF THE ALLOCATION PROBLEM

Introduction

In this chapter cost considerations associated with the improvement of availability are discussed, and a specific cost function is considered for detailed analysis and solution according to the set of solution equations obtained in Chapter III. Obviously, other cost relationships and factors could have been discussed and/or analyzed in detail for this development. However, it seemed more appropriate to follow through a complete solution for one form; so that this solution can be used, as it stands, for any particular problem to which the assumptions of Chapter III and the cost equation apply. Additional factors such as sparing costs, probabilities of meeting downtime goals, and total system costs, could be considered extensions of this work, and the techniques used in this paper could reasonably be applied to such extensions.

Cost Considerations of Availability Improvement

The overall objective of an allocation program should be to allocate the system parameters to the individual components in such a way as to minimize the costs of owning a system while meeting the system operational requirements. For the availability allocation problem, the costs of concern are those associated with the failure rates and repair times. Generally, these costs can be described by the following categories:

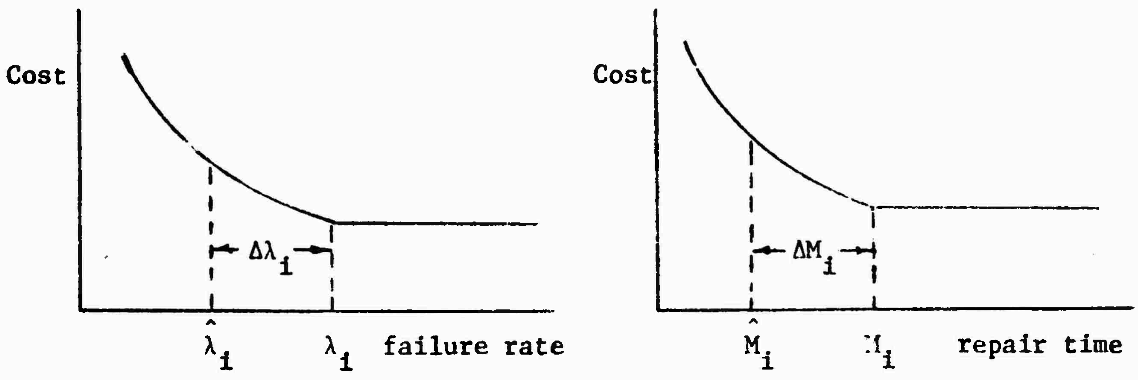
1. Sparing and inventory costs.
2. Downtime costs.
3. Development costs to increase the availability to the desired level.

The sparing and inventory costs are associated with the failure rates of a system. For instance, an item with a high failure rate would require more spare parts than a low failure rate item. Furthermore, if the failure rate of an item is decreased by one-half, then the number of spares necessary would also be decreased by approximately one-half. Likewise, the cost of carrying half as many spares in inventory, checking them out periodically and other related costs, would also be decreased by about one-half. Obviously for low failure rate items, where only a few spares were carried, this linear relationship would not hold. However, for a fairly large number of systems, with adequate inventory policies, the sparing cost would generally be expected to be a linear function of the failure rate.

The cost associated with system downtime is concerned with the period of time the system is out of operation. This cost would be a function of the frequency of failure (failure rate) and the time required to bring the system back to operating state (repair time). Such factors as lost output during repair time, manpower to perform repairs, and added burdens on the unfailed systems would be considered functions of the failure rates and repair times. If all hourly costs associated with the system downtime are lumped, it appears that the downtime cost could probably be considered a linear function of the failure rate and repair time.

Finally, a cost must be associated with actually achieving a particular availability. As stated in Chapter I, this paper is not concerned with trade-offs or procedures to establish an optimal availability requirement. It is assumed that an availability goal has been established and therefore, the problem is the achievement of that goal with the minimum

cost. Functions describing the cost of increasing the availability from a level below the requirement to the required goal should take the form of the curves shown in Figure 1.



(a) (b)

Legend:
λ_i achieved failure rate
$\hat{\lambda}_i$ improved failure rate

Legend:
M_i achieved repair time
$\hat{M}_i$ improved repair time

Figure 1. COST VERSUS REPAIR TIME AND FAILURE RATE

Figure 1-a shows a functional relationship between the failure rate and the cost required to improve it by $\Delta\lambda_i$. A function of this type would appear to be a good description of the relationship between the cost and the failure rate. The cost would vary inversely with the failure rate, such that a high cost would be associated with a low failure rate and a low cost with a high failure rate. Similar curves would exist for the repair times (Figure 1-b). These relationships suggest that the cost of a zero failure rate or repair time would be infinite since they are impossible to attain. Furthermore, the achieved level (λ_i or M_i) would represent a point above which the cost would not decrease. This means that since λ_i and M_i have already been obtained, no variable cost can be associated with getting to these points. Also, it is noted that if the function increased with the increasing failure rate past the achieved

level, λ_i, then the model would force the cost back to the achieved level. Thus the level portion of the cost curves in Figure 1 represents the case with increasing cost as well as the level cost associated with points past the achieved level. Therefore, the only areas of interest are between the acheived levels and the improved levels that will yield the availability requirement.

Nye (15) has proposed modified exponential forms that exhibit the above characteristics. However, his work determined that these were not amenable to analytical solution techniques.

For this paper, the following functions are used to describe the component improvement costs:

$$g_i\ (M_i,\ \Delta M_i) = \frac{Cm_i}{M_i + \Delta M_i} - \frac{Cm_i}{M_i} \quad , \tag{20}$$

$$h_i\ (\lambda_i,\ \Delta\lambda_i) = \frac{C\lambda_i}{\lambda_i + \Delta\lambda_i} - \frac{C\lambda_i}{\lambda_i} \tag{21}$$

where

1) $g_i\ (M_i,\ \Delta M_i)$ is the cost in dollars of improving the achieved repair time M_i by ΔM_i,
2) $h_i\ (\lambda_i,\ \Delta\lambda_i)$ is the cost in dollars of improving the achieved failure rate λ_i by $\Delta\lambda_i$,
3) Cm_i is the cost factor associated with the difficulty of decreasing the repair time of component i, and
4) $C\lambda_i$ is the cost factor associated with the difficulty of decreasing the failure rate of component i.

These functions follow the general characteristics described earlier. When the failure rate improvement, $\Delta\lambda_i$, becomes close to the achieved failure rate, λ_i, then the first term of Equation 21 becomes increasingly

large ($\Delta\lambda_i$ is negative). In addition, when $\Delta\lambda_i$ is zero (no improvement), the cost is zero. Thus, the achieved levels were 'free' in that no cost is incurred getting to that point. The cost comes about with improvements to meet the system availability requirement. The same is true for the repair time cost function.

The cost factors, Cm_i and $C\lambda_i$, are constants that are determined for each component based on past experience, familiarity with the equipment, and engineering judgment. The importance of careful estimates for these factors cannot be overemphasized. An example problem is stated and solved in Chapter V which illustrates the use of these factors. General guidelines for their establishment are discussed in the following paragraphs.

In the establishment of cost factors the relative values are the important consideration. For instance, if an item with a high failure rate could be improved easily by simple changes in the wiring system to give a much lower failure rate and other items could be improved only with great difficulty, then the cost factor $C\lambda_i$ might be one-tenth as great as some of the other cost factors. If one component is impossible to improve, then its cost factor might be set extremely high. Similarly for the repair time factors, estimates must be made of the relative degree of difficulty for decreasing component repair times.

The dimensions of $C\lambda_i$ and Cm_i are shown below:

$$\$ = \frac{C\lambda_i}{\text{failure/hr}} \quad \text{and therefore} \quad C\lambda_i = \$/\text{hr}$$

$$\$ = \frac{Cm_i}{\text{hr}} \quad \text{and therefore} \quad Cm_i = (\$)(\text{hr})$$

The failure rate cost factor is divided by the failures per hour, whereas the repair time cost factor is divided only by hours. Thus, the failure rate cost factor must be considerably lower than the repair time factor if

the costs associated with changing one or the other are to be similar. For example, if one component of a system has the following characteristics,

$$\lambda_i = .001/\text{hr}$$

$$M_i = 1 \text{ hr}$$

and it is determined that improvement effort can be made with equal difficulty for either characteristic, then the cost associated with each would be

$$TC_\lambda = \frac{C\lambda_i}{.001/\text{hr}}$$

and

$$TC_m = \frac{Cm_i}{\text{hr}} \quad .$$

Since the cost associated with the two is to be the same, then $C\lambda_i$ must be approximately one one-thousandth of Cm_i. When considering systems of more than one component, the cost factors must be balanced according to the degree of difficulty of improving the failure rate or repair time, as well as the relative difficulty of improving one component with respect to another.

In this paper the sparing and downtime costs are not considered in the analytical solution, since in the design stage of system development knowledge of these factors is somewhat limited. It is suggested, however, that when such costs can be predicted, these be included in the constants associated with the failure rate and repair time improvement. For instance, if it is known that one component costs twice as much as another, although improvement effort is comparable, the constant for the more expensive item should be increased, thereby making its "improvement" more costly. Similarly, if downtime is not exceptionally costly to a particular system,

all of the repair time constants could be lowered thus allowing less of the improvement to be taken by the failure rate.

Solution of the Allocation Problem

If the cost functions discussed in the previous section (Equations 20 and 21) are substituted into the solution set developed in Chapter III, the following set is obtained.

$$\frac{\partial}{\Delta\lambda_i}\left(\frac{C\lambda_i}{\lambda_i + \Delta\lambda_i} - \frac{C\lambda_i}{\lambda_i}\right) + K\,(M_i + \Delta M_i) = 0 \quad ,$$

for $i = 1, 2, \ldots, n$

$$\frac{\partial}{\Delta M_i}\left(\frac{Cm_i}{M_i + \Delta M_i} - \frac{Cm_i}{M_i}\right) + K\,(\lambda_i + \Delta\lambda_i) = 0 \quad ,$$

for $i = 1, 2, \ldots, n$

and

$$\Sigma M_i\,\Delta\lambda_i + \Sigma\Delta M_i\,\lambda_i + \Sigma\Delta\lambda_i\,\Delta M_i - \frac{A - A^*}{A^*A} = 0 \quad .$$

The solution to this set represents the set of improvements in component failure rates and repair times that minimize the improvement cost while achieving the availability requirement.

Taking derivatives yields

$$\frac{C\lambda_i}{(\lambda_i + \Delta\lambda_i)^2} = K\,(M_i + \Delta M_i) \qquad \text{for } i = 1, 2, \ldots, n$$

and

$$\frac{Cm_i}{(M_i + \Delta M_i)^2} = K\,(\lambda_i + \Delta\lambda_i) \qquad \text{for } i = 1, 2, \ldots, n \quad .$$

Solving these for the variables of interest, $\Delta\lambda_i$ and ΔM_i, one obtains

$$\Delta\lambda_i = \frac{\sqrt{C\lambda_i/K}}{\sqrt{M_i + \Delta M_i}} - \lambda_i \qquad \text{for } i = 1, 2, \ldots, n \tag{22}$$

$$\Delta M_i = \frac{\sqrt{Cm_i/K}}{\sqrt{\lambda_i + \Delta\lambda_i}} - M_i \qquad \text{for } i = 1, 2, \ldots, n \tag{23}$$

and

$$\Sigma M_i \, \Delta\lambda_i + \Sigma\Delta M_i \, \lambda_i + \Sigma\Delta\lambda_i \, \Delta M_i = \frac{A - A^*}{A^*A} \quad . \tag{24}$$

It is noticed that with the exception of the last equation the repair time improvement for each component, ΔM_i, is a function of the failure rate improvement $\Delta\lambda_i$ and K. Similarly, the failure rate improvement for any particular component is a function of ΔM_i and K. Thus, each set of two simultaneous equations, $\Delta\lambda_1$ and ΔM_1, $\Delta\lambda_2$ and ΔM_2, can be solved in terms of the Lagrange constant K. This is accomplished by first substituting $\Delta\lambda_i$ (Equation 22) into Equation 23 with the result that

$$\Delta M_i = \frac{\sqrt{Cm_i/K}}{\left(\lambda_i + \sqrt{\dfrac{C\lambda_i/K}{M_i + \Delta M_i}} - \lambda_i\right)^{1/2}} - M_i \quad .$$

This expression can be rewritten to give

$$\Delta M_i = \sqrt[3]{1/K}\left(\frac{Cm_i^{\,2}}{C\lambda_i}\right)^{1/3} - M_i \qquad \text{for } i = 1, 2, \ldots, n \tag{25}$$

Substituting for ΔM_i in Equation 22 yields the result

$$\Delta\lambda_i = \sqrt[3]{1/K}\left(\frac{C\lambda_i^{\,2}}{Cm_i}\right)^{1/3} - \lambda_i \qquad \text{for } i = 1, 2, \ldots, n \tag{26}$$

If Equations 25 and 26 are substituted for each $\Delta\lambda_i$ and ΔM_i, $i = 1, 2, \ldots, n$, in the constraint Equation 24, then the solution for $(1/K)^{1/3}$ is obtained as

$$(1/K)^{1/3} = \left[\frac{(A - A^*)/A^*A + \Sigma\lambda_i M_i}{\Sigma (Cm_i C\lambda_i)^{1/3}} \right]^{1/2} \tag{27}$$

Thus, the solution is obtained for each repair time and failure rate improvement. The constant $(1/K)^{1/3}$ is calculated and the result substituted in Equations 25 and 26 for ΔM_i and $\Delta\lambda_i$ to determine the repair time and failure rate improvement for each component. This solution represents the optimal set of improvements to meet the availability requirement A*.

An example problem is stated and solved in Chapter V using this result. In addition, the difficulty of the calculations, especially for a large number of components, has necessitated a computer program to aid in the rapid solution of the allocation problem. This program is also discussed in Chapter V.

Allocation in the Conceptual Stage of Design

During the conceptual stage of system development it is often possible to estimate costs of reliability and maintainability improvement, even though a definite design has not been formalized. If such costs can be estimated, however roughly, then an allocation program prior to the establishment of a detailed system design would offer several advantages. With a general concept of the failure rates and repair times that need to be achieved in order to meet a system availability requirement, the designer could proceed more logically and less expensively to the final design. Fewer design modifications would be necessary, and organization and direction would be given to the design effort.

Cost predictions for cases of this nature could not be as detailed as those of the case discussed earlier. In the previous case, a less than optimal set of conditions had been developed that fell short of the system goal. Then, cost functions were defined to change the conditions according to the least cost method to achieve the availability requirement. For the case discussed in this section, the solution will give the optimal set of component conditions at any availability level chosen. Thus, the cost of development will be optimized at all points as the availability is improved.

The cost model that is suggested for this case is a modification of the earlier model in that a cost has to be associated with any failure rate or repair time, and there are no achieved levels to be subtracted.

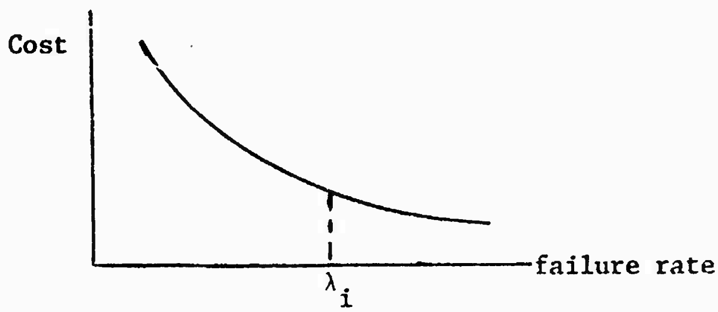


Figure 2. FAILURE RATE VERSUS COST FOR ALL LEVELS OF FAILURE RATE

Thus, using the reciprocal relations discussed in the previous section but redefining the variables of interest to be λ_i and M_i (rather than $\Delta\lambda_i$ and ΔM_i), the costs of failure rate and repair time development are

$$h_i\ (\lambda_i) = \frac{Cf_i}{\lambda_i} \tag{28}$$

and

$$g_i\ (M_i) = \frac{Cu_i}{M_i} \tag{29}$$

where Cf_i is the cost factor associated with the difficulty of developing

the failure rates and Cu_i is the cost factor associated with the difficulty of developing the repair times.

The cost factors in this case, Cf_i and Cu_i, are determined by the best possible predictions for each component. However, in the conceptual stage, a 'component' might be defined as a black box to perform a certain function, rather than as a definite piece of hardware. For example, a small electronics unit might be considered to consist of the following components: optics, circuit package, mechanical equipment, and tubes and transistors. Although detailed estimates of the failure rates for each 'component' could not be attained, it would be possible to predict that the reliability of the mechanical equipment could be improved at lower cost than the optics equipment. The tubes and transistors could be improved at lower cost than the circuits. Similarly, higher order systems could be analyzed for the necessary components and reasonable, relative cost factors determined.

The solution to this case is obtained as before by minimizing the development costs subject to the constraint of meeting an availability requirement, A*. The availability definition

$$A^* = \frac{1}{1 + \Sigma\lambda_i M_i}$$

is used to determine the constraint, since λ_i and M_i are the variables of interest in this case. Rewriting the above equation yields

$$\Sigma\lambda_i M_i = \frac{1}{A^*} - 1 \quad . \tag{30}$$

The total cost is obtained by summing Equations 28 and 29 over all components with the result that the total cost is given by

$$TC = \sum_{i=1}^{n} \frac{Cf_i}{\lambda_i} + \sum_{i=1}^{n} \frac{Cu_i}{M_i} \quad . \tag{31}$$

Using the Lagrange multiplier technique, one obtains

$$\Lambda = \sum \frac{Cf_i}{\lambda_i} + \sum \frac{Cu_i}{M_i} + K \left[\Sigma\lambda_i \, M_i - \frac{1}{A^*} + 1 \right]$$

$$\frac{\partial\Lambda}{\partial\lambda_i} = -\frac{Cf_i}{\lambda_i^2} + KM_i = 0 \qquad \text{for } i = 1, 2, \ldots, n$$

$$\frac{\partial\Lambda}{\partial M_i} = -\frac{Cu_i}{M_i^2} + K\lambda_i = 0 \qquad \text{for } i = 1, 2, \ldots, n$$

$$\frac{\partial\Lambda}{\partial K} = \Sigma\lambda_i \, M_i - \frac{1}{A^*} + 1 = 0$$

The solution to the above set of equations is obtained as in the previous case. The results are

$$M_i = (1/K)^{1/3} \left(\frac{Cu_i^2}{Cf_i} \right)^{1/3} \qquad \text{for } i = 1, 2, \ldots, n \quad , \tag{32}$$

$$\lambda_i = (1/K)^{1/3} \left(\frac{Cf_i^2}{Cu_i} \right)^{1/3} \qquad \text{for } i = 1, 2, \ldots, n \quad , \tag{33}$$

and

$$(1/K)^{1/3} = \left[\frac{1/A^* - 1}{\Sigma \, (Cu_i \; Cf_i)^{1/3}} \right]^{1/2} \quad . \tag{34}$$

The solutions obtained in this chapter are applied to an example problem in Chapter V. The computer program developed to handle the calculations is described and discussed. An additional complication is also discussed in which a maximum allowable repair time is imposed as an added constraint on the allocation problem.

CHAPTER V

APPLICATION OF THE ALLOCATION TECHNIQUE

General

This chapter uses results from the previous chapter to solve an example allocation problem. Since the calculations are somewhat difficult and tedious, a computer program was developed to perform the entire allocation procedure. The solution of the problem is discussed in the context of the computer program. The example system considered in this chapter consists of only five components, although the model and the computer program could be used to handle a system of any size. Of course, the dimension statements would have to be increased to the desired level. The program is listed in the Appendix.

Example

A system is proposed which consists of five components. Predictions for the failure rate and time to repair of each component are given in Table 1. The repair time includes all active portions of downtime. For instance, these might consist of isolation of the failed component, removal, repair and/or replacement, and check-out. Although the predicted failure rates and repair times of the last three components are the same, the components are not identical.

Table 1

COMPONENT REPAIR TIMES AND FAILURE RATES

Component	Repair Time (hr)	Failure rate (fail/hr)
1	15	.0019
2	8	.0051
3	10	.0034
4	10	.0034
5	10	.0034

Availability = 0.8538

The availability of the above system is calculated to be 0.8538 using Equation 5. The availability goal is 0.95.

The facts that can be ascertained about the relative costs of improving the failure rates and repair times of these components have led to the establishment of the cost factors in Table 2.

Table 2

COMPONENT COST FACTORS

Component	Cm ($-hrs)	Cλ ($/hr)
1	17,000	25.00
2	12,000	18.00
3	20,000	10.00
4	20,000	5.00
5	30,000	10.00

It is desired to achieve the availability requirement with the least possible improvement cost.

Using Equation 27 from Chapter IV the constant $(1/K)^{1/3}$ is calculated. This result is used to calculate the failure rate and repair time improvements according to Equations 25 and 26. The improvement is added to the original values (improvement means negative $\Delta\lambda_i$ or ΔM_i) and the results in Table 3 are obtained. The availability calculation yields the required 0.95.

Table 3

ALLOCATED REPAIR TIMES AND FAILURE RATES

Component	New Repair Time (hrs)	New Failure Rate (f/hr)
1	2.96	.00436
2	2.62	.00393
3	4.48	.00224
4	5.64	.00141
5	5.87	.00196

Availability = 0.9500

These calculations are not shown in detail since the computer program is used to perform all computations. The input data consists of the achieved repair times and failure rates and the cost factors from Tables 1 and 2. The input data format is given in Appendix. The output values in Table 3 are given in the print-out shown in Appendix. The total cost of these improvements is calculated to be $13,238.

In analyzing the results of this allocation, it is noted that the failure rate of component 1 was increased from 0.0019 to 0.00436, while the other components showed decreasing failure rates and repair times. This case, in which $\Delta\lambda_i$ is positive, arises when the model associates negative cost with increasing a failure rate or repair time. From Equation 21, the cost is reduced when a component's failure rate or repair time is increased. Thus, the total cost reflects the saving for increasing the failure rate of component 1.

Two alternatives exist when a situation of the above nature arises. First, it could be assumed that the failure rate of another component needed to be improved at the expense of component 1. This means that originally more expense than necessary was put into achieving the low failure rate of the first component, while a cheaper, more advantageous route to the availability requirement existed. Since that has already

transpired, if one of the other components could now be improved at the expense of the first, a significant improvement would be realized. For example, similar parts might be exchanged among items, redundant circuits rearranged, or any other mechanical change might be made that favored one component over the first. Similarly, if one of the repair times showed a positive ΔM_i, it might be possible to lower other repair times by changing the packaging arrangement to favor those items which the allocation procedure indicated to improve. One component could be moved closer to the surface for easier access, while the component with the positive ΔM_i could be moved into the center of the package.

The second alternative is to force the increased failure rate back to its original level. If this is done, and no other changes made, then the availability would be forced higher than the 0.95 required level. However, it is possible to mathematically constrain the failure rate or repair time to its achieved level and calculate a new optimum. To do this it is necessary to return to the solution set, Equations 25, 26, and 24 in Chapter IV. It is remembered that the solution for $(1/K)^{1/3}$ was obtained by substituting for each ΔM_i and $\Delta\lambda_i$ in Equation 24. However, if $\Delta\lambda_1$ is set equal to zero, then the constraint equation becomes

$$\sum_{i=2}^{n} M_i \, \Delta\lambda_i + \sum_{i=1}^{n} \Delta M_i \, \lambda_i + \sum_{i=2}^{n} \Delta\lambda_i \, \Delta M_i = \frac{A - A^*}{A^* A}$$

This can be generalized to the following result:

$$\sum_{\substack{\text{over } i \\ \text{where} \\ \Delta\lambda \neq 0}} M_i \, \Delta\lambda_i + \sum_{\substack{\text{over } i \\ \text{where} \\ \Delta M \neq 0}} \Delta M_i \, \lambda_i + \sum_{\substack{\text{over } i \\ \text{where} \\ \Delta\lambda \text{ and } \Delta M \neq 0}} \Delta\lambda_i \, \Delta M_i = \frac{A - A^*}{A^* A} \qquad (35)$$

The equation below is obtained by substituting for $\Delta\lambda_i$ and ΔM_i in Equation 35.

$$(1/K)^{2/3} \sum_{\substack{\text{over } i \\ \text{where} \\ \Delta\lambda_i \text{ or } \Delta M_i \neq 0}} (Cm_i C\lambda_i)^{1/3} + (1/K)^{1/2} \sum_{\substack{\text{over } i \\ \text{where} \\ \Delta\lambda_i = 0}} (Cm_i \lambda_i)^{1/2}$$

$$+ (1/K)^{1/2} \sum_{\substack{\text{over } i \\ \text{where} \\ \Delta M_i = 0}} (C\lambda_i M_i)^{1/2} - \sum_{i=1}^{n} \lambda_i M_i - \frac{A - A^*}{A^* A} = 0 \qquad (36)$$

Roots for (1/K) for Equation 36 are obtained by Newton's Method. The subroutine RTNI (p. 54) in the computer program performs the solution by Newton's Method for equations of the form $F(x) = 0$. The RTNI parameters consist of an initial guess of the root, the maximum allowable error, the maximum number of iterations, and a subprogram to calculate the function and derivative values. For any other problem, new parameters might be necessary to obtain an adequate solution. When (1/K) is determined, it is raised to the one-third power and the solutions for the $\Delta\lambda_i$ and ΔM_i which are not equal to zero are obtained from Equations 25 and 26. The procedure to perform this solution has been incorporated into the computer program.

Returning to the example problem, it is necessary to recompute the allocated values based on setting the positive $\Delta\lambda_1$ equal to zero. The results in Table 4 are obtained from the computer solution of the equations described above.

Table 4

FINAL COMPONENT ALLOCATIONS

Component	Repair Time (hrs)	Failure Rate (failure/hr)
1	3.10	.00190
2	2.74	.00411
3	4.69	.00234
4	5.90	.00148
5	6.14	.00205

Availability = 0.9529

Comparing these results to Table 3, the initial allocation, it is seen that by restoring the failure rate of the first component to its original value, the repair times and failure rates of the remaining components do not have to be improved quite as much. For instance, the repair time of the fourth component is now 5.9 hours rather than 5.64 hours originally. The total cost for achieving the above parameters is $18,809.

In analyzing these results to determine which changes to make in the system to achieve the availability requirement, it is necessary to study the system under consideration to see if the indicated changes are realistic and reasonable. The results of the allocation technique are guidelines to be taken into consideration, not hard and fast rules that must be followed under any circumstances.

An additional feature of the computer program is that the total cost of the changes is calculated and printed out. Also, if no components exhibit a positive $\Delta\lambda_i$ or ΔM_i, a message stating that the allocations are complete is printed, the parts of the program concerned with the new solution are skipped, and the program is terminated.

Maximum Allowable Repair Time

The case will often arise where a maximum allowable repair time is imposed on the operation of a particular system. It is possible that a

system could meet the availability requirement and still experience a failure that would result in a long downtime that could produce disastrous results. For example, mail-sorting equipment must be repaired within minutes of a failure to prevent a complete bottleneck and tie-up in the mail routing processes. Similarly, for transportation vehicles it would usually be desirable to keep repair times below certain levels to ensure meeting schedules with high probability. Also, any type of limited production 'readiness' weapons systems would be expected to have maximum allowable repair times.

This problem can be handled in the context of the allocation technique of this paper. If a maximum allowable repair time is imposed on the system, then it is obvious that all of the component repair times must be equal to or lower than the maximum. Thus, if any predicted repair time exceeds the maximum allowable, it is set equal to the maximum before the allocation technique is employed. The cost of this improvement is then added to the total cost of the changes involved by the allocation procedure. Then, cost factors are determined as usual, the allocation program is applied, and the results analyzed for optimal improvements. The final allocations in the computer output are the ones of interest.

It should be noted that this allocation procedure is concerned only with the mean repair times. No attempt has been made to describe the form of the repair time densities. For this reason, if a maximum allowable repair time is imposed, it would be necessary to consider possible distributional forms such that variations around the mean could be analyzed. The probabilities of exceeding the maximum allowable could be determined if distributional parameters can be predicted. Therefore, lacking information concerning the variations about the mean, it would appear wise to attempt to achieve repair times somewhat below the maximum allowable.

In the following chapter, the results of this work are summarized and recommendations for further study are given.

CHAPTER VI

SUMMARY AND CONCLUSIONS

A method for the cost-based allocation of the availability parameters, repair times and failure rates, has been developed. The technique is based on the minimization of improvement costs subject to the constraint of an imposed system availability requirement. The usefulness of this technique lies in its ability to determine the set of component repair time and failure rate improvements that can be made with the least cost to achieve a specified level of availability. The allocation problem is solved by the method of Lagrange multipliers, and a computer program has been developed to perform the allocation procedure.

This allocation technique is applicable to systems which can be described by a series model; that is, all components are necessary for proper system functioning. Extension to other models has not been considered in this paper, although it would appear feasible to solve the allocation problem for cases in which definite system models could be developed. For instance, an availability model could be defined for a system with redundant components. This model could then be used as the series model is used in this paper. Extensions of this nature would greatly expand the usefulness and application areas of the allocation problem.

It is also assumed that the individual components in the series configuration exhibit constant failure rates and that failures occur independently. The removal of these assumptions would generalize the allocation procedure and certainly make it more realistic. However, without the constant failure rate assumption, analytic solution techniques are usually unfeasible, if possible at all. The effects of various modes of failure could be investigated by careful analyzation and prediction of possible failure patterns, and subsequent determination of the effect of these on the system availability.

The cost equations used in this development are defined to describe the costs associated with the improvement of component failure rates and repair times from achieved levels. Thus the availability requirement is obtained in the manner that requires the least cost to be expended in improvement of design and equipment. Although this problem would be of great importance to design and development groups, it would appear that the allocations should be made on the basis of minimizing the cost of the system throughout its life. In this respect, the cost equations could be expanded to include the effects of component allocations on such factors as sparing costs, downtime costs, and other cost aspects of system ownership. The ultimate goal, of course, would be to allocate to the system components the levels of reliability and maintainability that would minimize the overall total system lifetime costs.

Specific work could also be done to define the effects of various provisioning policies, manpower levels, and repair facilities on the system and component repair times. The approach used in this paper was

to allocate only mean repair times, assuming that reasonable predictions could be made for active repair times from a basic design of the system. While the active repair times (associated with the 'inherent' availability) are of primary interest to the designer of a system, the long-range planning functions must consider the realities of logistics and administrative delays and manpower shortages. Even after a basic allocation program has been instigated, these factors should be taken into consideration before indicated changes are made in the system.

This paper also considers the allocation problem in which the designer is starting with no specific, achieved levels of failure rates and repair times. In this case, general cost equations are determined for the reliability and maintainability development of system elements, and the optimal combination of repair times and failure rates is determined. This is referred to in the paper as allocation in the conceptual stage of development. The use of this type of procedure should give the designer guidelines as to which areas to consider for detailed development and to which areas developmental effort can be best applied.

In conclusion, the results of this paper should be of primary interest to the basic design functions. Where it is necessary to achieve a stated inherent availability requirement, this allocation technique will direct attention to the specific components that need to be improved, and to the characteristics of each component, failure rates and/or repair times, that need to be improved. The inclusion of the more complicating factors of overall system performance and the factors of total system lifetime costs will add depth and thoroughness to the solution of the allocation problem.

APPENDIX

APPENDIX

A COMPUTER PROGRAM FOR THE AVAILABILITY ALLOCATION PROBLEM

The purpose of the program listed in this Appendix is to provide a computerized technique for solving the availability allocation problem.

To rewrite the program for a larger system it is necessary to make only two changes in the program. These are:

1) The variables in the DIMENSION statement, line 1, must be dimensioned as large as the number of components in the system, and

2) The number of components, N, must be stated at line 5.

The availability requirement, AA, is given in line 4. The input data format is listed on page 51. The significant variables for the program are listed below, and a program flowchart follows.

Variables

XM (I)	Repair time of i^{th} component
CM (I)	Repair time cost factor for i^{th} component
XF (I)	Failure rate of i^{th} component
CF (I)	Failure rate cost factor for i^{th} component
XMN (I)	Allocated repair time for i^{th} component
XFN (I)	Allocated failure rate for i^{th} component
A	Coefficient of $(1/K)^{2/3}$ in Equation 36

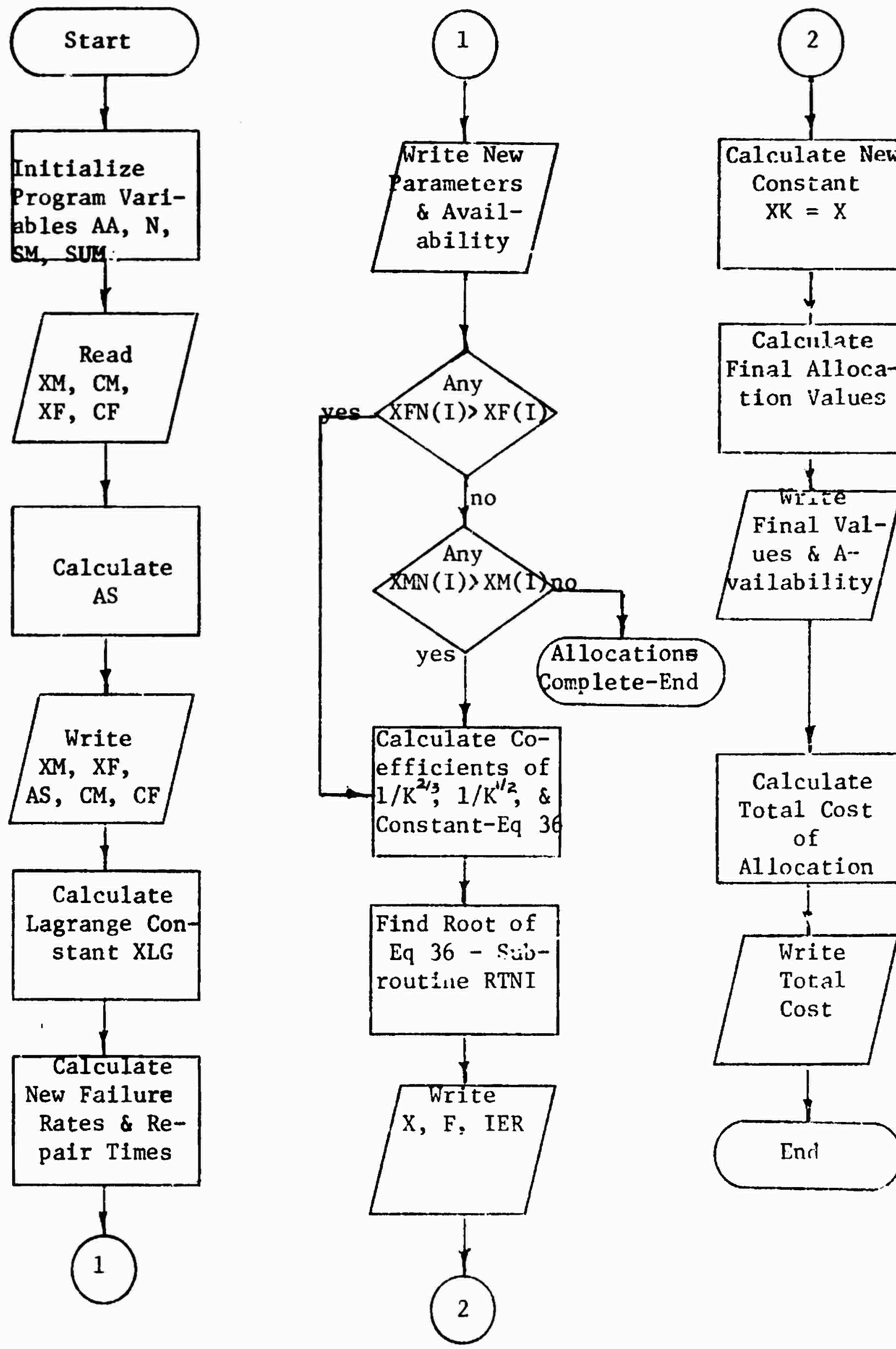
B	Sum of coefficients of $(1/K)^{1/2}$ in Equation 36
C	Constant terms in Equation 36
AA	Availability requirement
N	Number of components
AS	Achieved availability
XLG	Lagrange multiplier
ANEW	Availability from initial allocation
X	(1/K) from Equation 36, solution variable for RTNI
F	Functional value of Equation 36 when (1/K) equal to X
DERF	Value of first derivative of Equation 36 when (1/K) equal to X
AF	Availability based on final allocation
IER	Error code for subroutine RTNI

IER = 0 solution is obtained

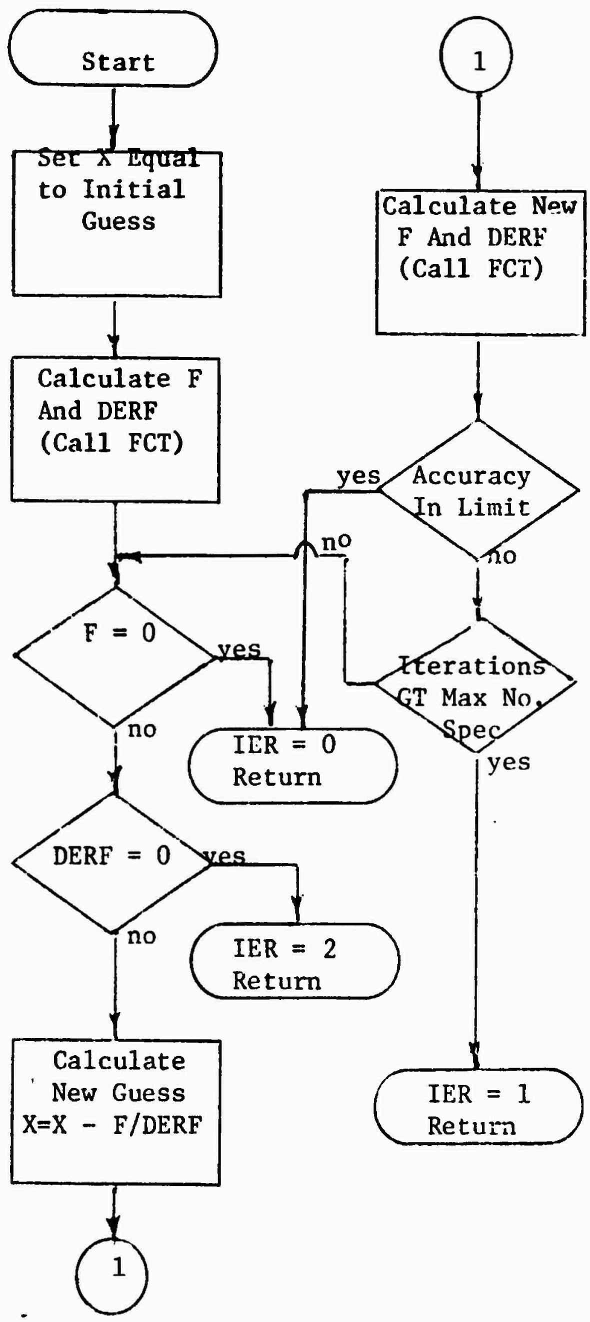
= 1 no convergence after maximum number of iteration steps

= 2 derivative equal to zero

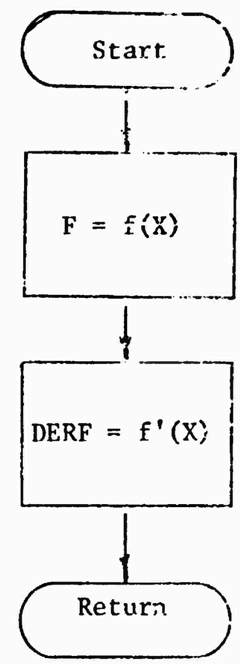
MAIN PROGRAM FLOWCHART



SUBROUTINE RTNI



SUBROUTINE FCT



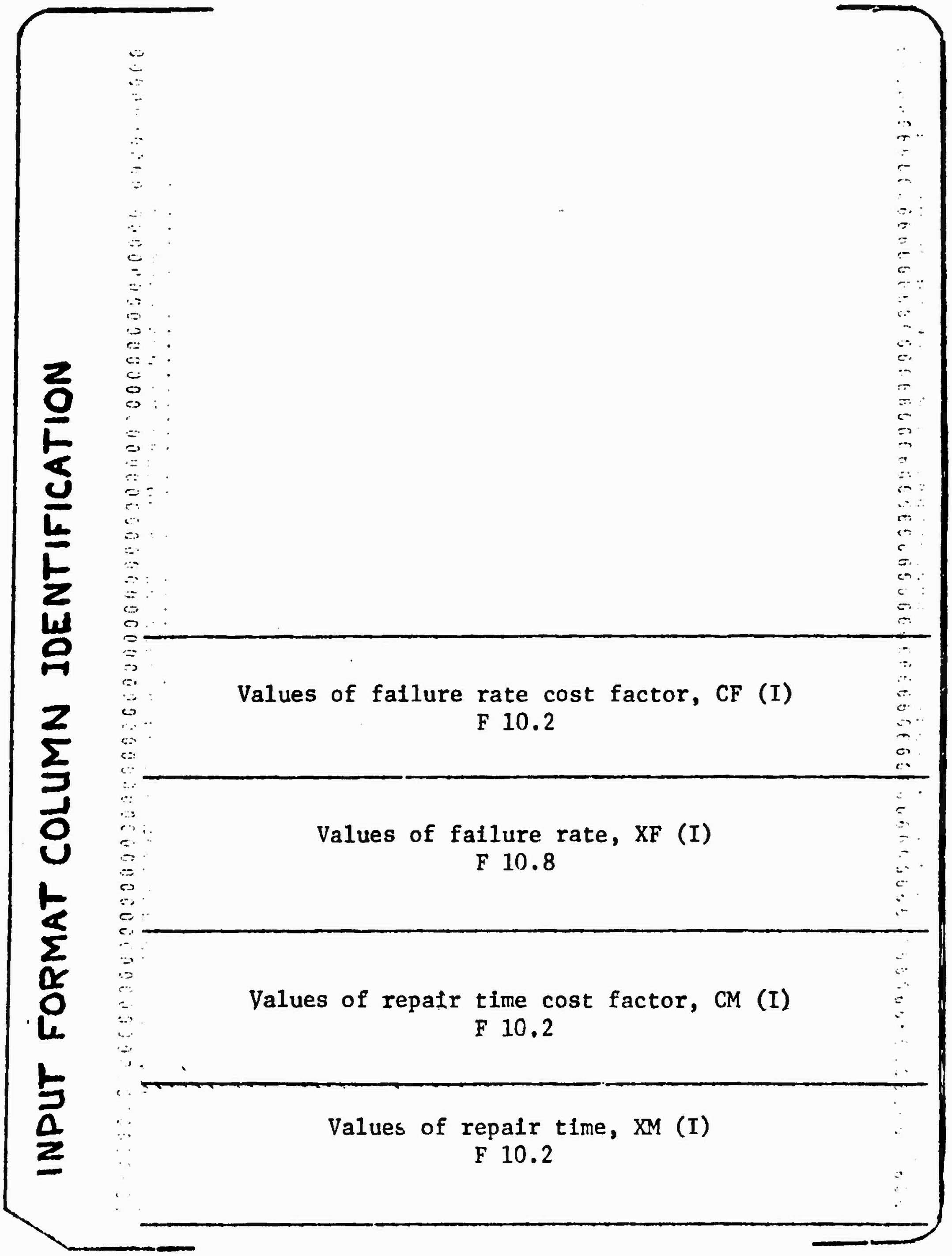


Figure A.1--Input format for the allocation program values

```
$JOB X41553,TIME=*20,PAGES=020  MESSER - ALLOCATION
      DIMENSION XM(16), CM(16),XF(16),CF(16),
     1XMN(16),XFN(16)
      EXTERNAL FCT
      COMMON A,B,C
      AA = .95
      N = 5
      SM = 0
      SUMC3 = 0
      SUM = 0
      DO 10 I = 1,N
      READ (5,90) XM(I), CM(I), XF(I), CF(I)
   90 FORMAT (4F10.0)
   10 CONTINUE
      WRITE (6,89) AA
   89 FORMAT (1H1,'AVAILABILITY REQUIREMENT IS ',F6.4////
     17X,'COMPONENT    REPAIR TIME (HRS)     FAILURE RATE')
      DO 20 I = 1,N
   20 SUMC3 = SUMC3 + (CM(I)*CF(I))**.33333
      XLGO = SQRT ((1.000/AA-1.000)/SUMC3)
      DO 30 I = 1,N
      XMN(I) = XLGO* CM(I)**.66667/CF(I)**.33333
      XFN(I) = XLGO*CF(I)**.6667/CM(I)**.3333
      WRITE(6,98) I, XM (I), XF (I)
   98 FORMAT (10X,I2,12X,F6.3,8X,F12.8)
      SUM = SUM + XMN(I) * XFN(I)
   30 CONTINUE
      AAC = 1.000/(1.000+SUM)
      SUMN = 0
      DO 40 I = 1, N
   40 SM = SM + XM(I)*XF(I)
      AS = 1.000/(1.000+SM)
      WRITE (6,91) AS
   91 FORMAT (1H0,'GIVEN ABOVE VALUES    A = ',F07.4///)
      WRITE (6,95)
   95 FORMAT  (21X,'COST FACTORS FOR ITEMS'/
     17X,'COMPONENT    REPAIR TIME    FAILURE RATE')
      DO 60 I = 1,N
      WRITE (6,96)  I,CM(I),CF(I)
   60 CONTINUE
   96 FORMAT (10X,I2,9X,F6.0,9X,F6.0)
      WRITE (6,94)
   94 FORMAT (1H0,//' INITIAL ALLOCATIONS ARE' /7X,
     1'COMPONENT    NEW REPAIR TIME (HRS)    NEW FAILURE RATE')
      XLG = SQRT(((AS - AA)/(AS*AA) + SM) / SUMC3)
      DO 50 I = 1,N
      XMN(I) = XLG * CM(I)**.66667/CF(I)**.33333
      XFN(I) = XLG *CF(I)**.6667/CM(I)**.3333
      WRITE (6,92) I, XMN(I),XFN(I)
   92 FORMAT (10X,I2,15X,F6.3,12X,F12.8)
      SUMN  = SUMN + XMN(I)*XFN(I)
   50 CONTINUE
      ANEW = 1.000/(1.000+SUMN)
      WRITE (6,99) ANEW
```

```
   99 FORMAT(1H ,25HAVAILABILITY CHECK   A = ,F07.4/////)
      DO 143 I = 1,N
      IF (XFN(I) - XF(I)) 144,144,146
  144 IF (XMN(I) - XM(I)) 143,143,146
  143 CONTINUE
      WRITE (6,962)
  962 FORMAT  (25H ALLOCATIONS ARE COMPLETE )
      GO TO 241
  146 A = 0
      B = 0
      C = SM + (AS - AA)/(AS*AA)
      DO 100 I = 1,N
      IF (XFN(I)-XF(I))  130,110,110
  110 IF (XMN(I) - XM(I)) 150,115,115
  115 C = C - XF(I)* XM(I)
      XMN(I) = XM(I)
      XFN(I) = XF(I)
      GO TO 100
  130 IF (XMN(I) - XM(I)) 190,160,160
  160 XMN(I) = XM(I)
      B = B+(CF(I)*XM(I))**.5000
      GO TO 100
  150 XFN(I) = XF(I)
      B = B+ (CM(I)*XF(I)) **.50000
      GO TO 100
  190 A = A + (CM(I) * CF(I)) ** .33333
  100 CONTINUE
      X = 0
      F = 0
      DERF = 0
      CALL RTNI (X,F,DERF,FCT, .00001, 1.00E-08, 25,IER)
      WRITE (6,888) X ,F,      IER
  888 FORMAT (1H ,'RTNI PARAMETERS   X  = ',F12.8,
     1'  F =',F12.8,'  IER =',I1/////
     2' FINAL COMPONENT ALLOCATIONS ARE '///X,
     3'COMPONENT   REPAIR TIME   FAILURE RATE ')
      XK = X **.33333
      SM = 0.
      DO 200 I = 1,N
      IF (XMN(I) - XM(I)) 210,220,220
  210 XMN(I) = XK * CM(I) **.66667 / CF(I)**.3333
  220 IF (XFN(I) - XF(I)) 230,200,200
  230 XFN(I) = XK       * CF(I)**.66667 / CM(I)** .33333
  200 CONTINUE
      DO 235 I = 1,N
  235 SM = SM + XFN(I) * XMN(I)
      AF = 1.000 / (1.000 + SM)
      DO 240 I = 1,N
      WRITE (6,93) I,XMN(I), XFN(I)
   93 FORMAT (10X,I2,9X,F6.3,5X,F12.8)
  240 CONTINUE
      WRITE (6,99) AF
```

```
      TC = 0.0
      DO 300 I = 1,N
      TC = TC +CM(I)/XMN(I) - CM(I)/XM(I)+
     1CF(I)/XFN(I) - CF(I)/XF(I)
  300 CONTINUE
      WRITE (6,889) TC
  889 FORMAT (1H ,'THE TOTAL COST IS   $',F10.2/////)
  241 A =0
      CALL EXIT
      END

      SUBROUTINE FCT (X,F,DERF)
      COMMON A,B,C
      IF (X.LE.0)    X = ABS(X)
      F = A*X**.66667 + B*X**.50000 - C
      DERF = .66667*A/X** .33333 + .50000*B/X** .50000
  301 RETURN
      END

      SUBROUTINE RTNI(X,F,DERF,FCT,XST,EPS,IEND,IER)
C     PREPARE ITERATION
      IER=0
      X=XST
      TOL=X
      CALL FCT(TOL,F,DERF)
      TOLF=100.*EPS
C     START ITERATION LOOP
      DO 6 I=1,IEND
      IF(F)1,7,1
C     EQUATION IS NOT SATISFIED BY X
    1 IF(DERF)2,8,2
C     ITERATION IS POSSIBLE
    2 DX=F/DERF
      X=X-DX
      TOL=X
      CALL FCT(TOL,F,DERF)
C     TEST ON SATISFACTORY ACCURACY
      TOL=EPS
      A=ABS(X)
      IF(A-1.)4,4,3
    3 TOL=TOL*A
    4 IF(ABS(DX)-TOL)5,5,6
    5 IF(ABS(F)-TOLF)7,7,6
    6 CONTINUE
C     END OF ITERATION LOOP
C     NO CONVERGENCE AFTER IEND ITERATION STEPS. ERROR RETURN.
      IER=1
    7 RETURN
C     ERROR RETURN IN CASE OF ZERO DIVISOR
    8 IER=2
      RETURN
      END
```

COMPONENT	REPAIR TIME (HRS)	FAILURE RATE
1	15.000	0.00190000
2	6.000	0.00510000
3	10.000	0.00340000
4	10.000	0.00340000
5	10.000	0.00340000

GIVEN ABOVE VALUES A = 0.3533

COST FACTORS FOR ITEMS

COMPONENT	REPAIR TIME	FAILURE RATE
1	17000.	25.
2	12000.	18.
3	20000.	10.
4	20000.	5.
5	30000.	10.

INITIAL ALLOCATIONS ARE

COMPONENT	NEW REPAIR TIME (HRS)	NEW FAILURE RATE
1	2.951	0.00435556
2	2.619	0.00392956
3	4.478	0.00223982
4	5.642	0.00141096
5	5.863	0.00195669

AVAILABILITY CHECK A = 0.9500

RTNI PARAMETERS X = 0.00000257 F = -0.00000004 IER =0

FINAL COMPONENT ALLOCATIONS ARE

COMPONENT	REPAIR TIME	FAILURE RATE
1	3.099	0.00190000
2	2.741	0.00411066
3	4.686	0.00234305
4	5.904	0.00147602
5	6.141	0.00204684

AVAILABILITY CHECK A = 0.9529

THE TOTAL COST IS $ 18809.34

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